LESSON 6.3b

Inverses of Logarithms

Today you will:

- Use inverse properties of logarithmic and exponential functions
- Graph logarithmic functions
- Practice using English to describe math processes and equations

Core Vocabulary:

- Logarithm base b of y, p. 310

Previous:
- Inverse functions

First, a few items of important review we will need for today…

Where is the “base” in an Exponential Function?

$$y = ab^{x}$$

Base

Where is the base in a Logarithm?

Base

$$\log_{b} y = x$$

What does it mean that one function is the inverse of another?

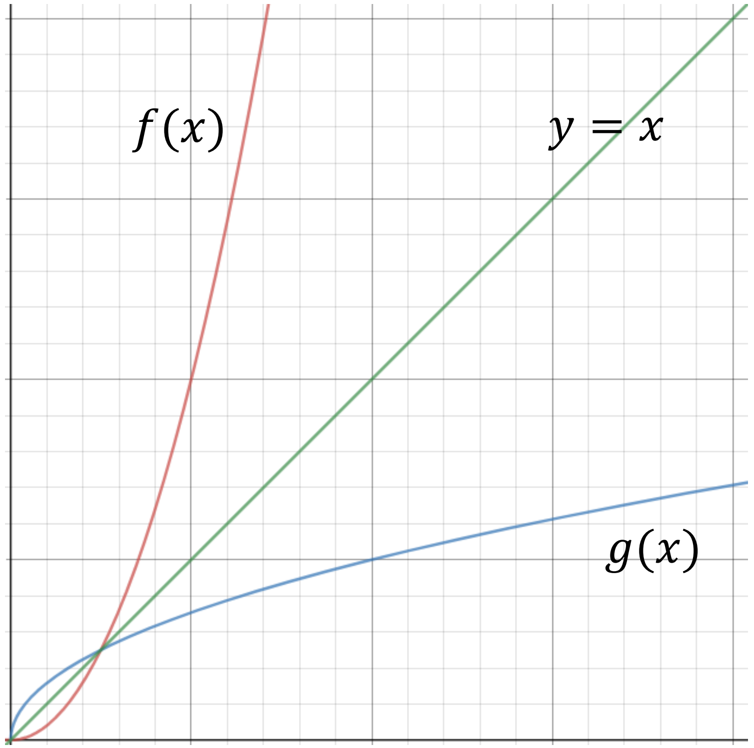
- They are "opposites"
- They "undo" each other
- Their graphs are reflections around the line $y = x$
- If you plug one into the other and simplify, you end up with just x

Example:

$$f(x) = 4x^2 \text{ and } g(x) = \tfrac{1}{2}\sqrt{x}$$

$$f(g(x)) = 4\left(\frac{1}{2}\sqrt{x}\right)^2 = 4\left(\frac{1}{4}x\right) = x$$

$$g(f(x)) = \frac{1}{2}\sqrt{4x^2} = \frac{1}{2}(2x) = x$$



Given $f(x) = \log_b x$ and $g(x) = b^x$
...find $f(g(x))$ and $g(f(x))$

This shows that $y = \log_b x$ and $y = b^x$ are inverses of each other!

$f(g(x)) = \log_b g(x)$	Write $f(x)$ plugging in $g(x)$
$y = \log_b b^x$	Rewrite as $y =$ and fill in what $g(x)$is
$b^y = b^x$	Rewrite in exponential form: $x = \log_b y \rightarrow y = b^x$
$y = x$	The statement can only be true if $y = x$

$g(f(x)) = b^{f(x)}$	Write $g(x)$ plugging in $f(x)$
$y = b^{\log_b x}$	Rewrite as $y =$ and fill in what $f(x)$ is
$\log_b y = \log_b x$	Rewrite in logarithmic form: $y = b^x \rightarrow x = \log_b y$
$y = x$	The statement can only be true if $y = x$

What does this mean?

If you see something in exponent form like this:

Log in the exponent

$$b^{\log_b x}$$

Both having the same base…

Example:

$3^{\log_3 18} = 18$

The answer is x

Why? Because this is $g\big(f(x)\big)$ from the prior slide.

Or if you see something in logarithmic form like this:

Log of an exponential

$$\log_b b^x$$

Both having the same base…

Example:

$\log_5 5^3 = 3$

The answer is x

Why? Because this is $f\left(g(x)\right)$ from the prior slide.

Simplify (a) $10^{\log 4}$ and (b) $\log_5 25^x$.

SOLUTION

a. $10^{\log 4} = 4$ — $b^{\log_b x} = x$

b. $\log_5 25^x = \log_5(5^2)^x$ — Express 25 as a power with base 5.

$= \log_5 5^{2x}$ — Power of a Power Property

$= 2x$ — $\log_b b^x = x$

How do you find the inverse of a logarithmic or exponential function?

Follow the normal process:

1. Rewrite as $y =$
2. Swap x and y
3. Solve for y
    - If in exponent form, convert to logarithmic form
    - If in log form, convert to exponential form

Find the inverse of each function.

a. $f(x) = 6^x$

b. $y = \ln(x + 3)$

SOLUTION

a. From the definition of logarithm, the inverse of $f(x) = 6^x$ is $g(x) = \log_6 x$.

b.

$y = \ln(x + 3)$ Write original function.

$x = \ln(y + 3)$ Switch x and y.

$e^x = y + 3$ Write in exponential form.

$e^x - 3 = y$ Subtract 3 from each side.

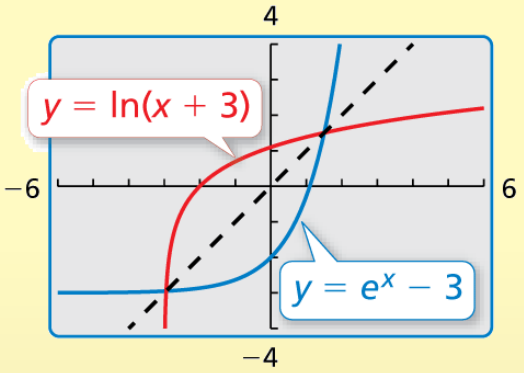
The inverse of $y = \ln(x + 3)$ is $y = e^x - 3$.

Check

a. $f(g(x)) = 6^{\log_6 x} = x$ ✓

$g(f(x)) = \log_6 6^x = x$ ✓

b.



The graphs appear to be reflections of each other in the line $y = x$. ✓

Homework

Pg 315, #35-54